PTL APPLICATION NOTE AN1

GENERATION OF ECT IMAGES FROM CAPACITANCE MEASUREMENTS

Issue 3

March 2001

SUMMARY

This note describes how the permittivity distribution of the material inside an ECT sensor is obtained from measurements of the capacitances between pairs of electrodes placed around the perimiter of the sensor using the Linear-Back-Projection method.

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GENERATION OF ECT IMAGES FROM CAPACITANCE MEASUREMENTS

1. INTRODUCTION

An ECT system provides a means for determining the distribution of a mixture of two dielectric (insulating) materials inside a vessel. It does this by measuring the capacitances between combinations of pairs of electrodes (**electrode-pairs**) placed around the perimiter of the sensor. The sensor cross-section can be of any shape, but most work to-date has concentrated on sensors having a circular cross-section.

For practical reasons, the number of electrodes placed around the sensor perimiter rarely exceeds 16 and values of 6,8 or 12 are more common. The maximum number of independent electrode-pair capacitance measurements is given by the equation:

$$M = E.(E - 1)/2 \qquad (1.1)$$

where E is the number of electrodes. For a 12-electrode sensor, there will be 66 independent electrode-pair capacitances.

The permittivity distribution can be defined in any convenient form. A common format, uses a square grid of 32 x 32 = 1024 pixels to represent the sensor cross-section.

An ECT system attempts to compute the permittivity distribution from the electrode pair capacitance measurements. With a 32 x 32 grid, the task can be seen to be to calculate the values of 1024 pixel elements from 66 capacitance measurements. It is mathematically impossible to carry out this calculation accurately as there is insufficient information available due to the limited number of electrode-pair capacitance measurements. The task therefore reduces to finding the best possible approximate solution to the problem. In the following sections we will attempt to explain how this is achieved in a practical ECT system.

The techniques used to solve this problem evolved over an extended period of time during the original research into ECT at UMIST and elsewhere. However, very little information apears to have been published about this aspect of the research work. What follows was uncovered by reading the original ECT papers and UMIST PhD theses, together with extensive discussions of the problem with some of the original and current UMIST researchers.

The UMIST researchers made several successive refinements during their quest to find a practical method which would allow reasonably accurate images to be constructed from the electrode-pair capacitance measurements. As the image construction concept is not particularly easy to understand, we will explain the technique by following the steps taken by the original researchers to arrive at the method which is in current use. We will do this by considering initially a very simple (and artificial) sensor model. Once the principles have been explained, we will move onto more realistic sensor models.

1.2 A NOTE ON THE FORMAT OF THE TEXT

In what follows, we have tried to keep the explanation as simple as possible. At the same time, we want to lead the reader to the final solution adopted. We have tried to do this by indicating some sections in square brackets [ ]. These sections can be ignored on a first reading of the text. However, they will need to be read and understood ultimately to allow the reader to follow the mathematics introduced later.

2. SIMPLE SQUARE SENSOR MODEL

An ECT sensor is normally used to measure the permittivity distribution of a mixture of two dielectric materials. For simplicity in what follows, we shall assume that the lower permittivity material is air (ie the sensor is empty).

It is convenient to work in normalised units for both the permittivities and electrode-pair capacitances. Using this system, we define the normalised permittivity of the lower permittivity material to be represented by the value zero and the normalised permittivity of the higher permittivity material to be represented by the value 1.

We shall start by considering a very simple square capacitance sensor having 12 electrodes, with 3 electrodes located along each side of the square. The cross-section inside the sensor is represented by a grid of 3 X 3 = 9 pixels, labelled A to I, as shown in figure 1.

For simplicity, we shall assume that the electric field lines run only parallel to the sides of the square as shown in figure 2. This is a gross simplification of what happens in reality, but is a useful starting point in our attempt to explain how an ECT sensor works. We will show later how the principle can be extended for use with more realistic electric field distributions.

The capacitance between any pair of electrodes will only be affected by a pixel inside this sensor if the pixel intercepts the electric field lines between these electrodes. It follows that, for this simple sensor model, only those electrodes which are opposite each other on opposing sides of the sensor will be affected by changes in the permittivity of the pixels inside the sensor.

For this simple sensor model, we therefore need to consider only the capacitances between 6 electrode-pair combinations, namely:

1 - 9, 2 - 8, 3 - 7, 4 - 12, 5 - 11, and 6 - 10

We designate these capacitances as C_{1-9} etc. where C_{1-9} means the capacitance between electrodes 1 and 9 etc.

The capacitances measured between these electrode-pairs are also normalised in a similar manner to those of the permittivity values. When the sensor contains the lower permittivity material, we define the value of the normalised capacitances measured between the electrode pairs to be zero and to be 1 when the sensor is filled with the higher permittivity material.

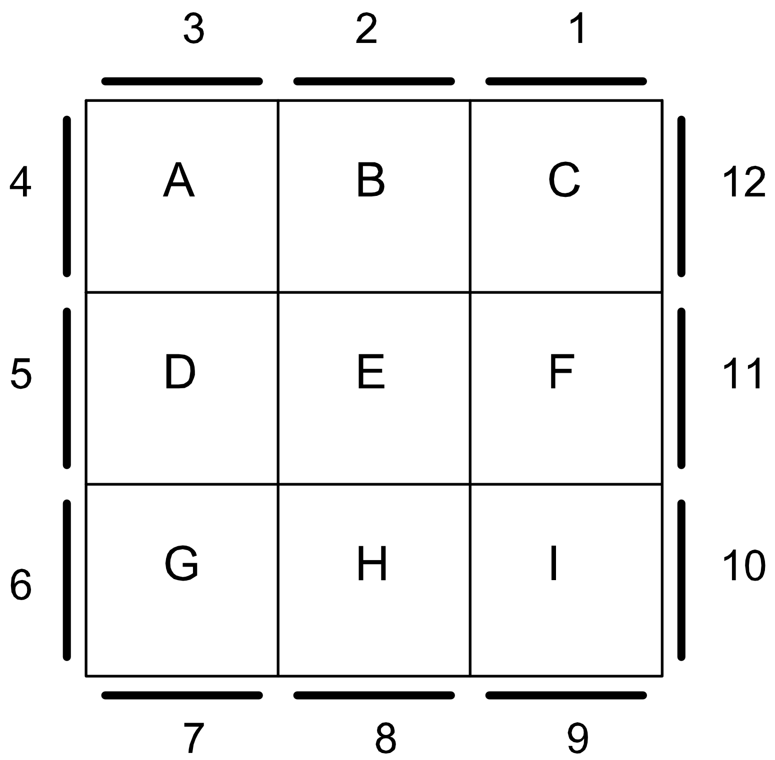

Figure 1. 12 - electrode square ECT sensor

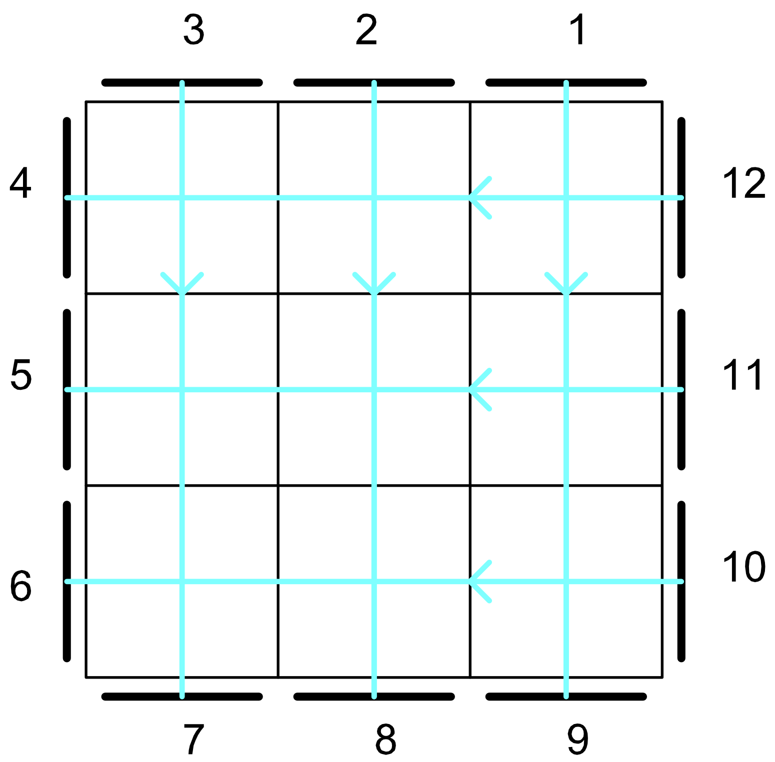

Figure 2. Simplified Electric field distribution

3. THE SENSITIVITY MAP OF THE SENSOR

We are going to consider how the capacitances measured between the electrode pairs of an empty sensor are affected when a dielectric probe, consisting of a square dielectric rod whose cross-section is the same size as one pixel, is located at each of the 9 pixel locations in turn.

By our previous definition, the normalised capacitances measured between any two pairs of electrodes will be zero when the sensor is empty.

We will first consider what happens when the dielectric probe is located at Pixel E in the otherwise empty sensor. This situation is represented in figure 3, where the red colour indicates that the centre pixel E is occupied by material having the higher value of permittivity (the dielectric bar) and the dark blue colour indicates that the remaining pixels are occupied by lower permittivity material (air).

In our simple sensor model, we have assumed that the electric field lines are parallel to the sides of the sensor, and it follows that the dielectric probe will only affect the capacitance measured between electrodes whose electric field lines interact with the cross section of the probe. Hence, in this case, the only electrode pairs whose capacitance will be affected will be electrode pairs 2-8 and 5-11.

If the probe is now moved to pixel B, the electrode pairs whose capacitance is changed will be electrodes 2-8 and 4-12. The same arguments can be used to determine which electrode pairs will be affected when the probe is placed in any of the other 7 pixels.

By locating the permittivity probe at each pixel location in turn, we can construct a 3 X 3 element matrix, for each electrode pair combination. These are known as sensitivity matrices, and show, for each electrode-pair, how the permittivity probe will affect the capacitance between the electrodes for each pixel location.

In the case of a 3 X 3 pixel grid, the sensitivity matrix for each electrode-pair consists of a set of 9 sensitivity coefficients corresponding to each of the pixels inside the sensor. The sensitivity matrix can be written as follows:

$$\begin{matrix} S_A & S_B & S_C \\ S_D & S_E & S_F \\ S_G & S_H & S_I \end{matrix}$$

where S_A, S_B etc. are the sensitivity coefficients for pixels A,B etc.

We will now further simplify the situation by representing the sensitivity coefficients in binary format. That is, if the probe affects the electrode pair capacitance at a particular pixel location, we signify this by entering the value 1 in the sensitivity matrix for this pixel location. Similarly, if there is no effect, we illustrate this by writing 0 in the pixel location.

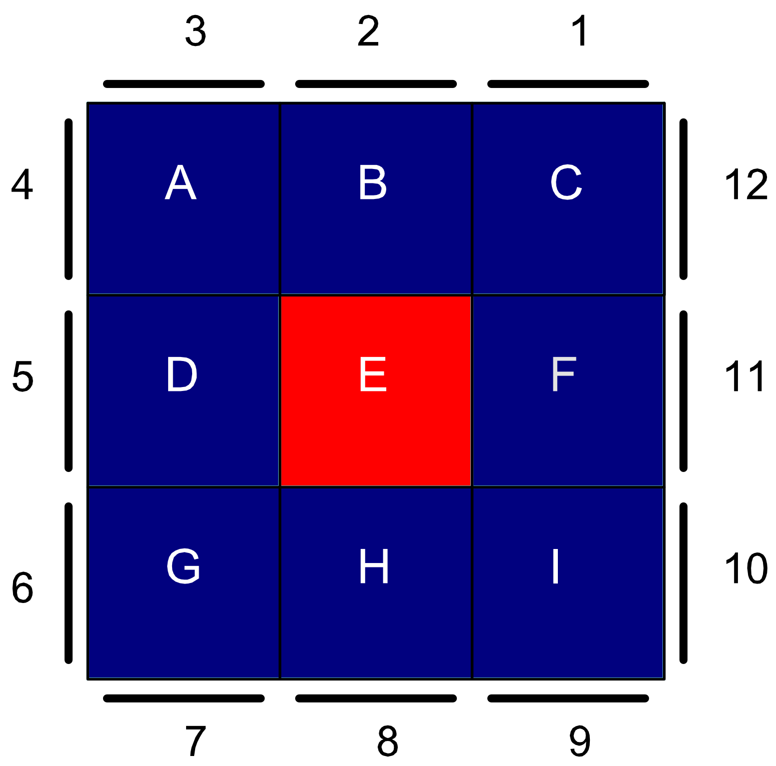

Figure 3. Pixel E containing high-permittivity matarial

This is best illustrated by some examples.

Consider the electrode pair combination $S_{2\text{-}8.}$ Bearing in mind our previous (highly simplified) assumption about the paths of the electric field lines inside the sensor, the sensitivity matrix for this electrode pair can be written down by inspection as follows:

$S_{2\text{-}8}$	0	1	0
	0	1	0
	0	1	0

Similarly, the matrix for the electrode pair 5-11 can be written:

$S_{5\text{-}11}$	0	0	0
	1	1	1
	0	0	0

All of the other sensitvity matrices can be written down in a similar manner.

Note that, for this sensor model, only 4 other electrode combinations will have matrices containing non-zero sensitivity coefficients (for electrode pairs 3 -7, 4 -12, 5 -11 and 6 - 10). All of the matrices for the remaining electrode pair combinations will contain only zeroes. This is because we have assumed that there is no capacitive coupling between any other electrode combinations, which in turn, follows from the simple electric field paths which we have assumed for this sensor.

The six sensitivity matrices which contain non-zero sensitivity coefficients are shown in diagrammatic form in figure 4. The white pixels represent sensitivity coefficients with zero coefficients and the dark pixels represent sensitivity coefficients with the value 1.

To summarise, each electrode-pair combination will have an associated sensitivity matrix containing N elements, where N is the number of pixels. The elements in the sensitivity matrix for each electrode pair indicate whether a change in the permittivity of a single pixel inside the sensor will affect the capacitance measured between the electrodes of this pair. In our first, very simple sensor model, if the capacitance changes when the permittivity of a particular pixel is increased or decreased, the pixel sensitivity coefficient for that pixel is assigned the value 1. If there is no change to the electrode-pair capacitance, then the pixel sensitivity coefficient is set to zero.

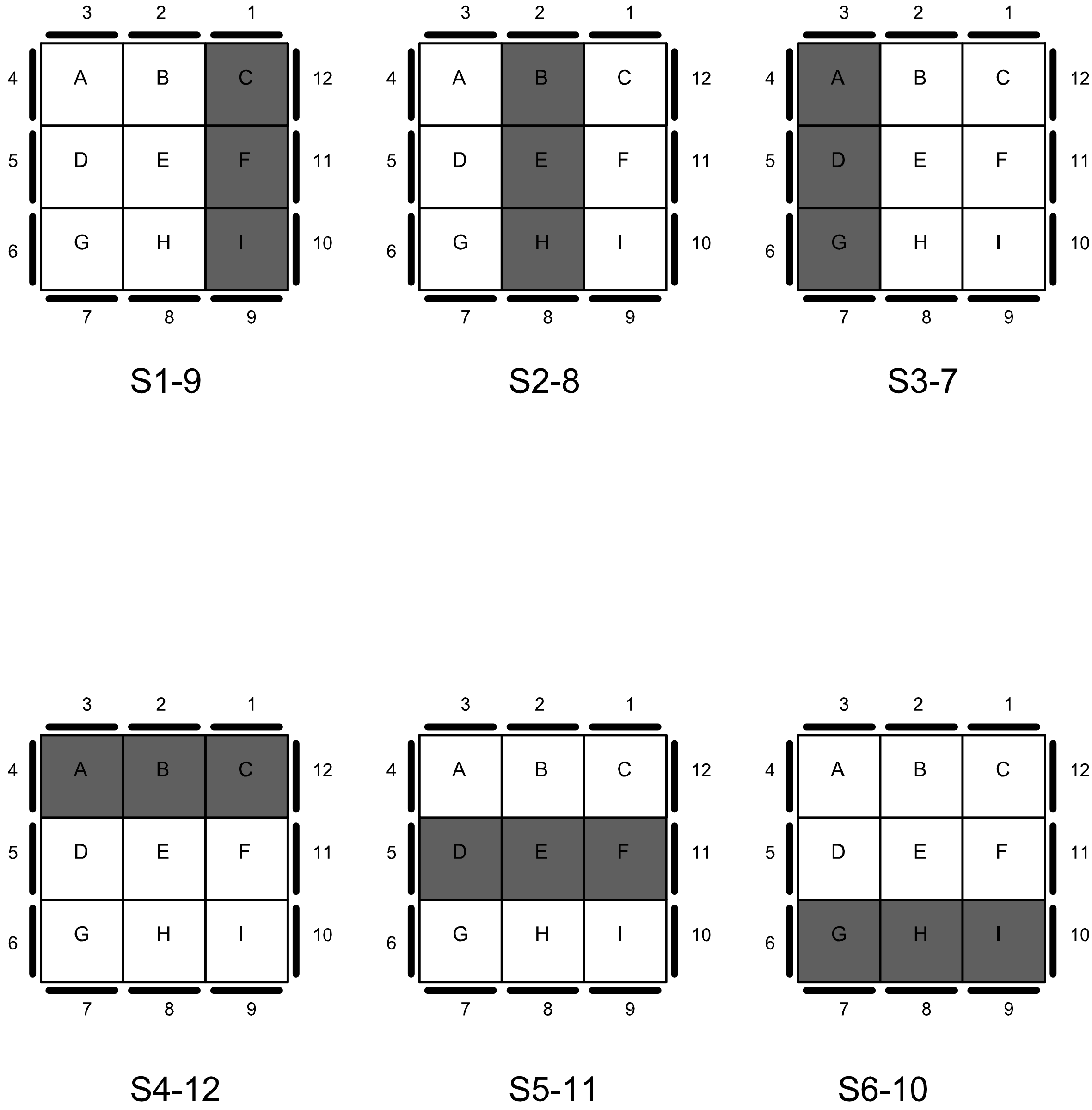

Figure 4. Simplified binary sensitivity maps for square ECT sensor

4. THE FORWARD PROBLEM

The next question to consider is how the electrode-pair capacitances change when an arbitary permittivity distribution exists inside the sensor.

We will assume that the sensor is filled with a mixture of the lower and upper permittivity materials. This will give rise to a set of normalised capacitance values for the set of electrode pairs and we will assume that these normalised capacitances will have values somewhere within the range 0 to 1.

The first task is to determine how to calculate the inter-electrode capacitances from the values of the normalised permittivities of the pixels for any distribution of permittivity inside the sensor. This is known as the **Forward problem**.

The method which is adopted to solve the **Forward problem** is based on the Electrical Network **Superposition Theorem.** Expressing this theorem in a form which is relevant to the forward problem, the Superposition Theorem states that, in a linear electrical network, the combined effect of a large number of stimuli can be found by adding up the effects which result when each stimulus acts individually.

In our case the **stimuli** are the **permittivity values** of the individual **pixels** inside the sensor and the **effects** are the **inter-electrode capacitances** measured between the **electrode-pairs**.

We can now apply this theorem to find the **capacitances between the electrode pairs** when the sensor is **filled with the higher permittivity material** as follows:

For each electrode-pair, we measure the set of elemental normalised capacitances between the electrodes when each pixel in turn (the probe pixel) contains the higher permittivity material, while all of the remaining pixels contain the lower permittivity material. For our simple square sensor with 9 pixels, there will obviously be 9 elemental capacitances to measure for each location of the probe pixel.

To find the capacitance between these two electrodes when the sensor is completely filled with the higher permittivity material, the Superposition theorem states that we should simply add up all of these 9 elemental capacitances.

For our simple sensor model, we know that for any particular electrode-pair, only some of the pixels inside the sensor will change the capacitance between this pair of electrodes and this information is contained in the sensitivity matrix. For the simple binary sensitivity matrix which we have discussed so far, the matrix tells us which pixels influence the capacitance between a given pair of electrodes and which pixels have no effect on this capacitance.

Applying the **Superposition theorem**, the capacitance C_m between any pair of electrodes can therefore be written in the following general form:

$$C_m = P_m.(S_A.K_A + S_B.K_B + S_C.K_C + \dots\dots\dots\dots S_I.K_I) \qquad (4.1)$$

Where:

m represents the electrode pair combinations (1-9, 2-8 etc.) and C_m is the (normalised) inter-electrode capacitance for the mth electrode-pair.

S_A, S_B, etc are the sensitivity coefficients of each individual pixel in the grid for the mth electrode pair.

K_A, K_B etc. are the values of (normalised) permittivity of the material in each pixel.

P_m is a constant included to ensure that the normalising remains valid, whose value can be determined as described below.

Now consider specifically the capacitance between electrodes 2 and 8 when the sensor is full of the higher permittivity material. In this situation, the normalised permittivity of each pixel (K_A, K_B etc.) will, by definition, have the value 1.

In the case of the square sensor, we have already shown that the sensitivity coefficients S_B, S_E and S_H have the value 1 while all of the remaining sensitivity coefficients for this electrode combination are zero.

In this case, equation 4.1 becomes

$$C_{2\text{-}8} = P_{2\text{-}8}.(1.1 + 1.1 + 1.1) = 3.P_{2\text{-}8} \qquad (4.2)$$

However, we also know that, by definition, the normalised electrode-pair capacitances for all electrode pairs have the value 1 when the sensor is full of the higher permittivity material. It follows that the value of the normalising constant $P_{2\text{-}8}$ for electrode-pair 2-8 must have the value 1/3.

We can calculate the value of P_m for all of the other electrode pair combinations in a similar manner. For the simple sensor model which we are currently using, it is clear that all values of P_m have the value 1/3. However, this simple result will not generally hold for more complex field distributions.

For any other values of pixel permittivity, we can find the electrode-pair capacitances by substituting the known values of normalised permittivity into equation 4.1 as follows:

eg for electrode-pair 2-8

$$C_{2\text{-}8} = (1/3).(1.K_B + 1.K_E + 1.K_H) \qquad (4.3)$$

For example, if the normalised pixel permittivities S_B, S_E and S_H are reduced to values of 0.5, then the capacitance C_{2-8} calculated from equation 4.3 will have the value:

$$C_{2-8} = (1/3).(0.5 + 0.5 + 0.5) = 0.5 \qquad (4.4)$$

As a second example, let us assume that the normalised pixels have different values e.g. let K_B =1, K_E =0.5 and K_H = 0.3.

In this case, C_{2-8} will have the normalised capacitance given by:

$$C_{2-8} = (1/3).(1 + 0.5 + 0.3) = 0.6. \qquad (4.5)$$

So far, we have based our model on the use of binary sensitivity coefficients, in the interests of simplicity. However, as will be shown later, if the true values of the sensitivity cofefficients can be found (either by calculation or by measurement), these can be used instead of the crude binary coefficients which we have so-far assumed.

It turns out that by using these more accurate sensitivity coefficients, the solution to the forward problem can be found with reasonable accuracy. However, we will stay with the binary sensitivity coefficients for the present for reasons which will become clear in the following sections.

[5. THE GENERAL SOLUTION OF THE FORWARD PROBLEM

Using equation 4.1, we can use the general expression for C_m to find, for example, the value of $C_{2\text{-}8}$ as follows:

$$C_{2\text{-}8} = P_{2\text{-}8}.(S_A.K_A + S_B.K_B + S_C.K_C + S_D.K_D + S_E.K_E + S_F.K_F + S_G.K_G + S_H.K_H \quad +S_I.K_I) \qquad (5.1)$$

There will of course be similar equations for the other electrode pairs. For example,

$$C_{1\text{-}9} = P_{1\text{-}9}.(S_A.K_A + S_B.K_B + S_C.K_C + S_D.K_C + S_E.K_E + S_F.K_F + S_G.K_G + S_H.K_H \quad +S_I.K_I) \qquad (5.2)$$

The complete set of relationships for all electrode pairs can be written concisely in matrix form as follows:

$$\mathbf{C} = \mathbf{S}.\mathbf{K} \qquad (5.3)$$

where **:**

C is an **M x 1** matrix containing the normalised electrode-pair capacitances C_m (in the nominal range 0 to 1).

M is the number of unique electrode-pair combinations (eg 66 for a 12-electrode sensor)

Each m value corresponds to an electrode - pair combination. For example, m=1 might be defined to be the pair 1-9, m = 2 to be the pair 2-8 etc.

K is an **N x 1** matrix containing the normalised pixel permittivities (in the nominal range 0 to 1)

N is the number of pixels representing the sensor cross-section (eg 1024 for a 32 x 32 grid)

S is an **M x N** matrix containing the set of sensitivity matrices for each electrode-pair. The matrix **S** is commonly referred to as the **sensitivity map** of the sensor.

Note that in this equation, the matrix **S** incorporates the normalising constants P mentioned previously. That is, the sensitivity coefficients are now normalised as well as the vales of C and K.

This matrix equation defines the general solution to the **forward problem**.]

6. THE INVERSE PROBLEM

As we have seen, once the set of sensitivity matrices (the sensitivity map) for the sensor has been found, it is a relatively straightforward process to calculate what the electrode-pair capacitances will be for a given permittivity distribution inside the sensor.

However, in an ECT system we normally want to be able to determine the **permittivity distribution inside the sensor** from knowledge of the **capacitances between the pairs of electrodes** which surround the sensor, ie the **inverse** of the problem which we have just outlined.

The question which therefore needs to be answered is, given a set of (measured) normalised electrode-pair capacitances, what are the values of normalised permittivity of each of the pixels inside the sensor which have given rise to this set of electrode-pair capacitances? This is known as the **inverse problem**.

We will seek to find a solution to the **inverse problem** using the sensitivity matrices which we have derived above. The method which we shall use is called **Linear Back-Projection (LBP)**.

The basis of the method is to consider each electrode-pair capacitance in turn and to consider which pixels inside the sensor have contributed to the change in the normalised capacitance from the zero values when the sensor is empty.

We do this by referring to the binary **sensitivity matrix** for each **unique electrode-pair**. Consider electrode-pair **2-8** for example. We know from the previous sections that only pixels B, E and H can have contributed to any change in the capacitance $C_{2\text{-}8}$. However, **we have no means of knowing how much of the change in capacitance to attribute to each pixel.**

Faced with this problem, **the LBP method makes the pragmatic asumption that, for a specific electrode-pair, all of the pixels with non-zero sensitivity coefficients contribute equally to the capacitance change for each specific electrode pair and that they do this by changing their permittivity values by identical amounts. It further assumes that these incremental changes in the permittivities of the pixels which contributed to this capacitance change are proportional to the change in the electrode-pair capacitance.** That is:

$$\Delta K_B = \Delta K_E = \Delta K_H = Q_n.C_{2\text{-}8} \qquad (6.1)$$

where ΔK_B is the elemental change in the normalised permittiviy of pixel B etc., $C_{2\text{-}8}$ is the normalised capacitance measured between electrode pairs 2 and 8 and Q_n is another scaling constant which is required to ensure that the normalising remains valid.

This situation is represented in figure 5 for electrode-pair $C_{2\text{-}8}$, which shows, in light-blue, the pixels which are assumed to have contributed to the capacitance between this pair of electrodes. Figure 6 similarly shows the pixels which contribute to the capacitance between electrodes 5 and 11.

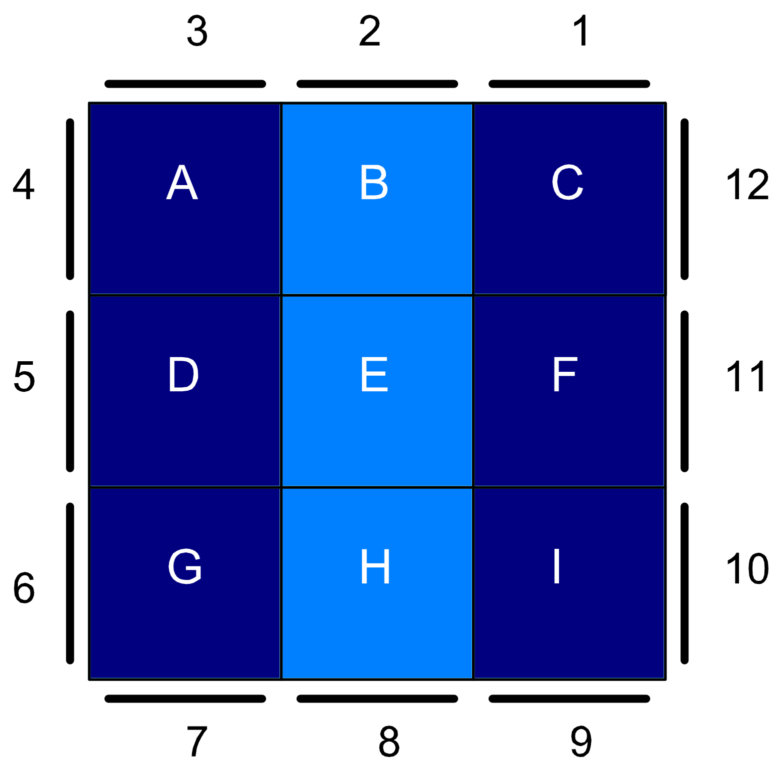

Figure 5. Elemental pixel values due to C2-8

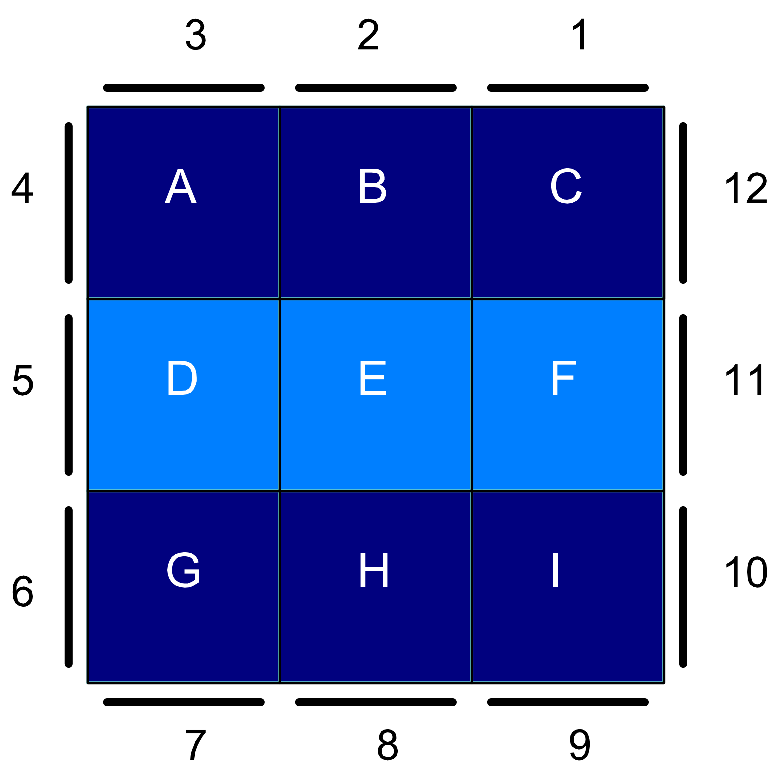

Figure 6. Elemental pixel values due to C5-11

[Equation 6.1 is a special case of the more general expression which needs to be written if we make no assumptions about the values of the sensitivity coefficients for each pixel. This general expression can be written as:

$$\Delta K_n = Q_n.S_{nm}.C_m \qquad (6.2)$$

where:

ΔK_n is the elemental change in permittivity of pixel n due to the capacitance C_m of electrode-pair m

Q_n is a normalising constant

S_{mn} is the sensitivity coefficient which corresponds to the nth pixel and the mth electrode-pair capacitance.]

The next assumption is that this process can be applied for each electrode-pair in turn and that the overall value of pixel permittivity for each individual pixel can again be found by applying the **Superposition theorem**.

Hence we obtain the pixel permittivity by summing the elemental values of permittivity obtained from the equations for each electrode pair.

For our simple square sensor, this can be expressed as follows, using pixel B as an example.

$$K_B = \Delta K_{B(1\text{-}9)} + \Delta K_{B(2\text{-}8)} + \Delta K_{B(3\text{-}7)} + \Delta K_{B(4\text{-}12)} + \Delta K_{B(5\text{-}11)} + \Delta K_{B(6\text{-}10)} \qquad (6.3)$$

Where $\Delta K_{B(1\text{-}9)}$ means the elemental change in permittivity of pixel B due to the capacitance change of electrode-pair 1-9 etc. Using equation 6.1 or 6.2 we obtain:

$$K_B = Q_B.(S_{B(1\text{-}9)}.C_{(1\text{-}9)} + S_{B(2\text{-}8)}.C_{(2\text{-}8)} + S_{B(3\text{-}7)}.C_{(3\text{-}7)} + S_{B(4\text{-}12)}.C_{(4\text{-}12)} + S_{B(5\text{-}11)}.C_{(5\text{-}11)} + S_{B(6\text{-}10)}.C_{(6\text{-}10)}) \qquad (6.4)$$

where $S_{B(1\text{-}9)}$ is the sensitivity coefficient of pixel B for electrode-pair 1-9, $C_{(1\text{-}9)}$ is the normalised capacitance between electrode pair 1-9 etc. and Q_B is a normalising constant for the permittivity of pixel B.

By inspection of figures 1 and 2, pixel B will only affect the capacitance between electrode-pairs 2-8 and 4-12.

Hence $S_{B(2\text{-}8)} = 1$ and $S_{B(4\text{-}12)} = 1$, while all of the other values of S_B are zero, giving:

$$K_B = Q_B.(1 + 1) = 2.Q_B \qquad (6.5)$$

We know that when all of the electrode-pair capacitances are 1, the normalised capacitances also have the value 1.

Substituting these conditions in equation 6.5 we obtain:

$$1 = Q_B.(2) \qquad (6.6)$$

and hence the value of $Q_B = 1/2$.

Similar expressions can be written for each of the other pixels.

For example, the permittivity of pixel A is given by:

$$K_A = \Delta K_{A(1\text{-}9)} + \Delta K_{A(2\text{-}8)} + \Delta K_{A(3\text{-}7)} + \Delta K_{A(4\text{-}12)} + \Delta K_{A(5\text{-}11)} + \Delta K_{A(6\text{-}10)} \qquad (6.7)$$

$$= Q_A.(S_{A(1\text{-}9)}.C_{(1\text{-}9)} + S_{A(2\text{-}8)}.C_{(2\text{-}8)} + S_{A(3\text{-}7)}.C_{(3\text{-}7)} + S_{A(4\text{-}12)}.C_{(4\text{-}12)} + S_{A(5\text{-}11)}.C_{(5\text{-}11)}$$
$$+ S_{A(6\text{-}10)}.C_{(6\text{-}10)}) \qquad (6.8)$$

By similar reasoning, it is clear that the value of Q_A is also 1/2. It turns out that the values of all of the normalising constants for the simple square sensor are also 1/2.

[In general terms, the permittivity of an individual pixel is given by:

$$K_n = \sum_{m=1}^{M} Q_n.S_{nm}.C_m \qquad (6.9)$$

where

K_n is the permittivity of pixel n
C_m are the set of normalised capacitance measurements for the M electrode pairs.
Q_n are the set of normalising constants for the pixel n.
S_{nm} are the set of sensitivity coefficients for pixel n and the M sets of electrode-pairs.

This can be written in matrix format as follows:

$$\mathbf{K} = \mathbf{S^T.C} \qquad (6.10)$$

where

K is an **N x 1** matrix containing the normalised permittivities of each of the **N** pixels.
$\mathbf{S^T}$ is the normalised transpose sensitivity map (formed by interchanging the rows and columns of the sensitivity map matrix **S)**
C is an **M x 1** matrix containing the **M** normalised electrode-pair capacitances.

Equation 6.10 is the general equation which is used in the LBP algorithm to solve the **inverse problem** and hence to calculate the **pixel permittivity distribution**.]

7. A SIMPLE EXAMPLE OF LINEAR BACK-PROJECTION

Having explained the principle of **Linear Back-Projection**, we will now consider a simple example.

Let's assume that the sensor contains a dielectric bar of square cross-section and normalised permittivity 1, which occupies pixel E in the centre of the sensor, as shown in figure 3. All of the other pixels will be assumed to contain material of normalised permittivity zero.

We have represented this situation in figure 3 using the colour scale which is used widely to display permittivity images in ECT systems. In this system, the upper value of normalised permittivity (1) is shown in red and the lower value (0) is shown in dark-blue. Intermediate permittivity values are shown on a scale which extends from blue through green to red.

We know from the sensitivity matrices, that pixel E will only affect the capacitance between electrode-pairs 2-8 and 5-11.

Using our knowledge of the sensitivity coefficients and the values of pixel permittivities, we can first calculate what the values of $C_{2\text{-}8}$ and $C_{5\text{-}11}$ will be using equation 4.1 as follows:

$$C_{2\text{-}8} = (1/3).\ (1.1) = 1/3 \qquad (7.1)$$

similarly

$$C_{5\text{-}11} = (1/3).(1.1) = 1/3 \qquad (7.2)$$

We can now apply the equation of back-projection (6.10) to re-calculate the value of the pixels from the electrode-pair capacitances.

Taking pixel A as an example, and bearing in mind that the only non-zero electrode-pair capacitances generated by the "dielectric bar" in pixel E are $C_{(2\text{-}8)}$ and $C_{(5\text{-}11)}$, we can calculate the value of pixel A as follows:

$$K_A = (1/2).\ (S_{A(2\text{-}8)}\ .C_{(2\text{-}8)} + S_{A(5\text{-}11)}.C_{(5\text{-}11)}). \qquad 7.3)$$

However, both $S_{A(2\text{-}8)}$ and $S_{A(5\text{-}11)}$ are zero for these two electrode-pair combinations.

Hence the value of K_A calculated using the LBP method is zero, which is correct, as the only pixel occupied by an object of non-zero permittivity is pixel E.

Using the obvious symmetry properties of the sensor, it is clear that similar results will be obtained for pixels C, G and I, and hence K_C, K_G and K_I will all have zero values.

This is a promising start, as we know from figure 3 that the actual pixel permittivities for pixels A, C, G and D are zero.

We will now repeat this process for pixel B.

$$K_B = (1/2).\ (S_{B(2\text{-}8)}\ .C_{(2\text{-}8)} + S_{B(5\text{-}11)}.C_{(5\text{-}11)}). \qquad (7.4)$$

In this case, $S_{B(2\text{-}8)}$ has the value 1 while $S_{A(5\text{-}11)}$ is zero for these two electrode-pair combinations.

Hence the value of K_B calculated by the LBP method is (1/2).(1/3) = 1/6.

Again, using the symmetry properties of the sensor, it is clear that the values of pixels D, F and H will be identical to that of pixel B, ie K_D, K_F and K_H will all have the value 1/6.

By inspecting figure 3, we know that these values should be zero. However, in this case, the LBP algorithm has assigned pixels B, D, F and H a finite positive value of 1/6.

We will now complete the process by calculating the value of pixel E.

$$K_E = (1/2).\ (SE_{(2\text{-}8)}\ .C_{(2\text{-}8)} + SE_{(5\text{-}11)}.C_{(5\text{-}11)}) \qquad (7.5)$$

In this case, both $S_{B(2\text{-}8)}$ and $S_{A(5\text{-}11)}$ have the value 1 for these two electrode-pair combinations.

Hence the value of K_E calculated by the LBP method is (1/2).((1/3) + (1/3)) = 1/3.

Referring back to figure 3, we know that the value of K_E should be 1. The LBP algorithm has therefore produced a value for pixel E which is significantly lower than its known value.

The results for the calculated pixel permittivites (in matrix format) can be summarised as follows:

0	1/6	0
1/6	1/3	1/6
0	1/6	0

These results are represented in figure 7, which represents pixel values of 0 as dark blue, 1/6 as light blue and 1/3 as green.

However, we know that the correct answer is the following matrix:

0	0	0
0	1	0
0	0	0

And this situation is shown in figure 3 on the same colour scale.

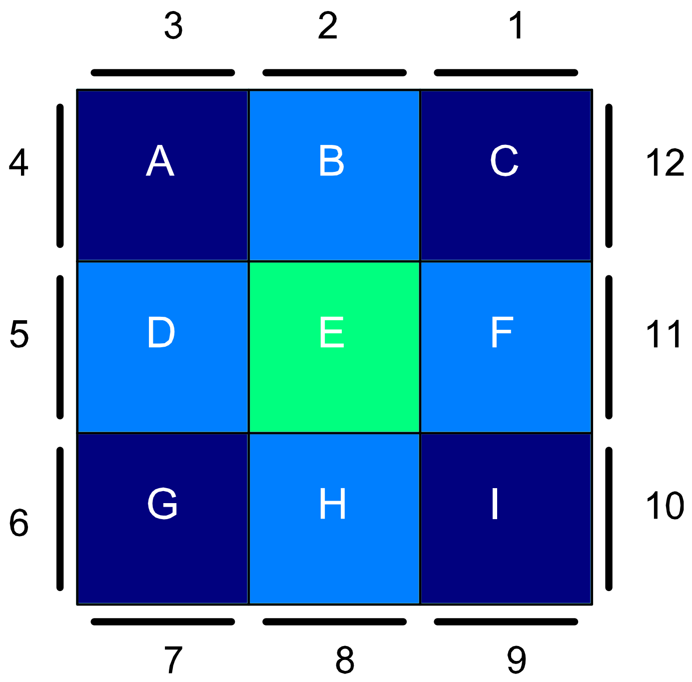

Figure 7. Composite pixel values from sums of elemental values

We have attempted to indicate how this permittivity distribution has occurred by showing the elemental pixel permittivities which result from the only two non-zero capacitance pair measurements $C_{2\text{-}8}$ and $C_{5\text{-}11}$ in figures 5 and 6.

Figure 5 shows the elemental pixel permittivities which result from the $C_{2\text{-}8}$ capacitance and figure 6 shows the elemental pixel permittivities which result from the $C_{5\text{-}11}$ capacitance. In these figures, the dark blue pixels have zero values and the light blue squares have normalised permittivities of 1/6 as calculated above.

When we add up these two permittivity distributions, we obtain figure 7, where the green pixel (E) has the value 1/3.

By comparing the known permittivity distribution (figure 3) with the results calculated using the LBP method, (figure 7), it is clear that what has happened is that the LBP algorithm has spread out, or blurred the original probe pixel over a much larger area than that occupied by the original pixel probe.

7.1 INHERENT CHARACTERICTICS AND ERRORS OF THE LBP METHOD

By comparing the LBP solution (figure 7) with the correct answer (figure 3), we can now identify some of the main characteristics of the LBP algorithm:

1. The calculated value of the probe pixel is 1/3 of its known value. In general, the LBP algorithm will always under-estimate areas of high permittivity.

2. Some of the pixels which should have zero values have had finite values assigned to them. In general, the LBP algorithm will over-estimate areas of low permittivity.

3. The sum of the calculated pixel permittivities equals that of the original single probe pixel. In general, the average permittivity of all of the pixels calculated by the LBP algorithm will approximately equal that of the sensor when it contains the test object. The LBP algorithm is therefore useful for calculating the average voidages or volume ratios of the sensor contents.

In practice, the use of a very simple sensor model and binary sensitivity coefficients have aggravated the deficiences of the LBP algorithm and this is therefore a rather extreme example. However, the results obtained here illustrate the principle and also the main failings of the LBP algorithm.

8. EXTENSION TO CIRCULAR SENSORS

Until now, we have considered a simple and rather artificial square sensor with a limited number of pixels and very restricted electric field lines. We will now show how the method can be extended to a circular sensor containing a much larger number of pixels and where we will not make any unrealistic assumptions about the electric field lines and inter-electrode coupling. For the case of the circular sensor, we will assume that there is capacitive coupling between all pairs of electrodes.

8.1 8-ELECTRODE CIRCULAR SENSOR

Consider the 8-electrode circular sensor shown in figure 8. The sensor is shown superimposed on a grid of 32 X 32 square pixels.

The approximate binary sensitivity maps of the sensor can be deduced by considering the paths of the Electric field lines between the electrode pairs. For example, the binary sensitivity map for pairs 1-2 is shown in figure 9(a). The black pixels indicate that the pixel will influence the capacitance between that electrode pair and the white pixels indicate that the pixel will have no influence on the capacitance between those pairs.

The binary sensitivity maps for electrodes pairs 1-3, 1-4 and 1-5 are shown in figures 9(b) to 9(d). It is obvious that the sensitivity maps for any other electrode-pair combination will be identical to one of these four basic maps (although rotated around the sensor), because of the symmetry properties of the sensor.

8.2 APPLICATION OF LINEAR BACK-PROJECTION

The principle of back-projection for circular sensors remains the same as that outlined in sections 6 and 7 for the simple square sensor. For each inter-electrode capacitance measurement, using the simple binary sensitivity maps, we assume that all pixels which have a non-zero sensitivity coefficient contribute equally to the measured change in capacitance between the electrode pairs.

Hence, for each electrode pair measurement, we allocate each pixel which has a non-zero sensitivity coefficient an elemental value which is proportional to the measured change in the normalised capacitance between these electrodes. We then add up the elemental values for the pixel for all of the electrode-pair capacitance measurements to obtain the value of the pixel permittivity.

The final step is to normalise this procedure so that all of the pixels have the value 1 when the normalised inter-electrode pair capacitances are all 1 and have the value 0 when all of the normalised inter-electrode pair capacitances are zero.

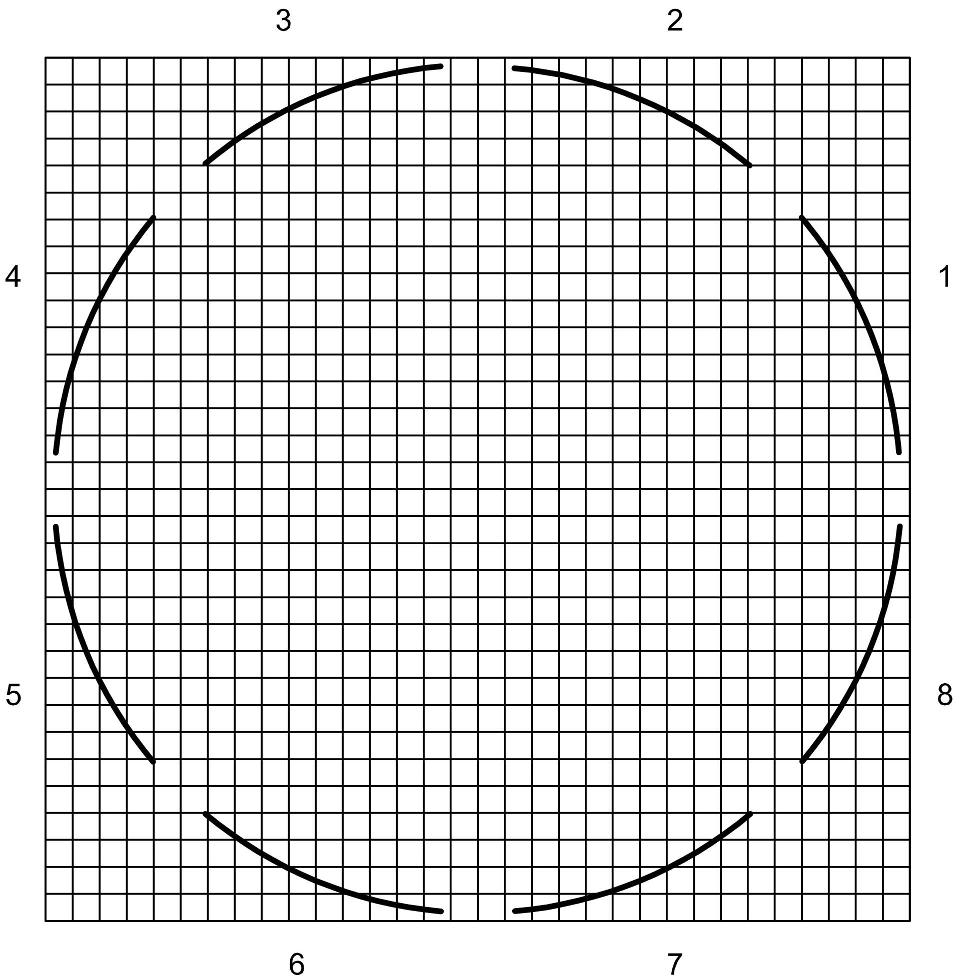

Figure 8.   8 - electrode circular sensor and 32 X 32 pixel grid

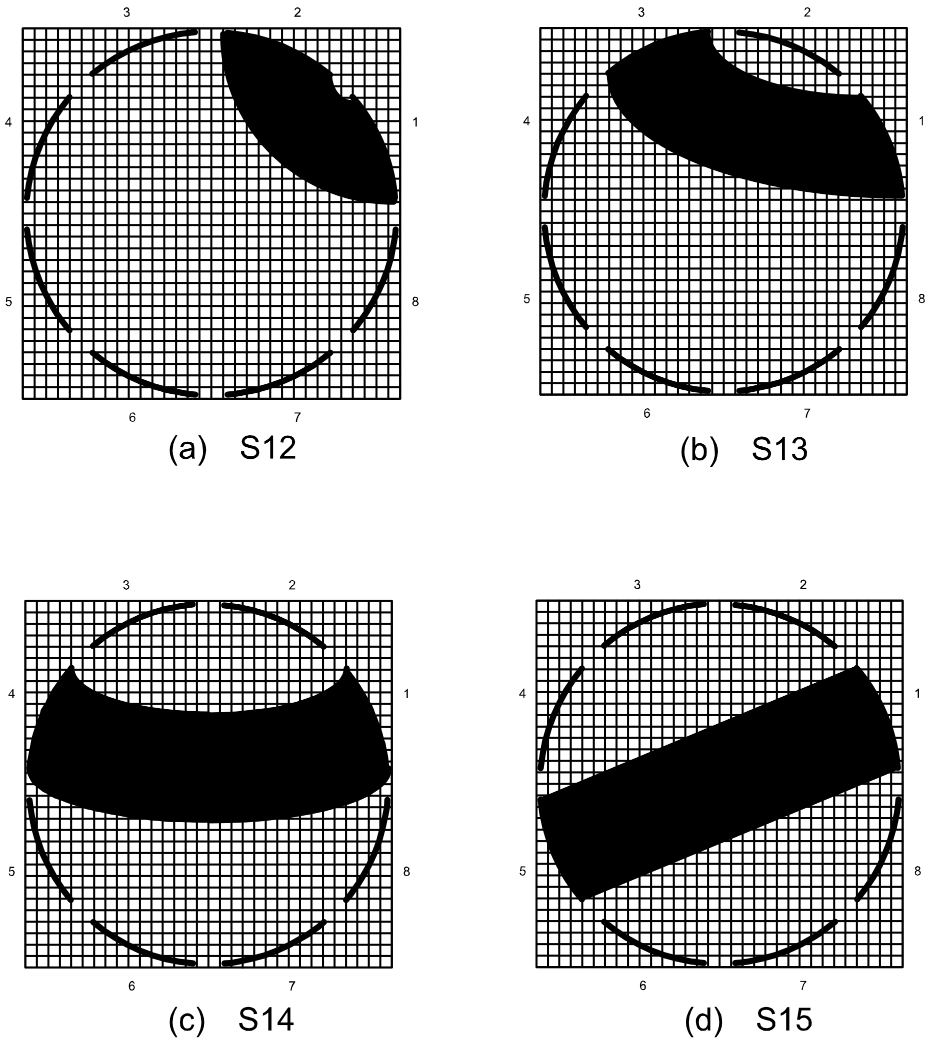

Figure 9. Binary sensitivity maps for 8 - electrode sensor

8.3 THE USE OF REAL SENSITIVITY MAPS

The LBP method can be easily extended for the more practical case where the sensitivity coefficients are not simply binary, but have values which lie within the nominal range 0 to 1. In this case, the elemental pixel values are found by multiplying the sensitivity coefficients by the normalised capacitance measurements using equations 6.9 and 6.10.

Some typical sensitivity matrices for an 8-electrode circular ECT sensor are shown in figure 10.

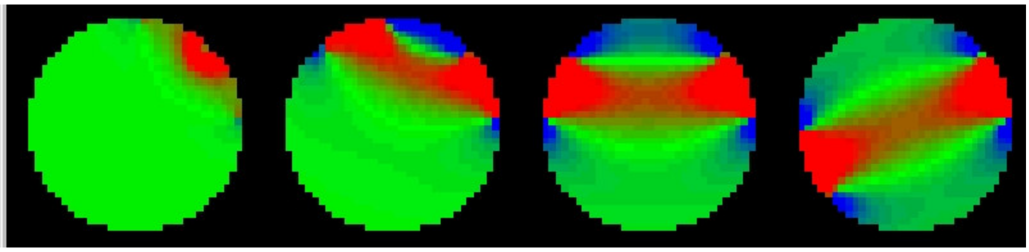

Figure 10. Primary sensitivity maps for an 8-electrode sensor.

We can therefore apply equation 6.10 directly to obtain the permittivity values of each pixel from the set of capacitance measurements.

8.3 CHARACTERISTIC RESULTS

From the description of the LBP method given in section 8.2 and previously, it is clear that images produced by this method will always be approximate. The method spreads the true image over the whole of the sensor area and consequently produces very blurred images. Moreover, because the image has been spread out over the sensor area, the magnitude of the pixels will always be less than the true values. However, the sum of all of the image pixels will approximate to the true value .

A typical image for a circular tube, containing the same material used to calibrate the sensor is shown in figure 11. The actual tube diameter was 40mm and the sensor internal diameter was 100mm. This corresponds to a nominal voidage of 16%. The LBP algorithm gave an overall voidage of between 12 and 16%, depending on the sensor model which was used*. The effects of image blurring are clearly visible. The correct image would be a filled red circle of diameter 40% of that of the sensor, where red corresponds to a normalised permittivity of 1. The central area of the actual image produced by back-projection has a much lower value of permittivity, around 0.5 and displayed as green in figure 11.

It is possible to improve this image using simple iterative techniques. One method for doing this is described in PTL application note AN4 (An iterative method for improving ECT images). An example of the improvement in image quality which can be obtained using this method is shown in figure 12 for the same measurement data as shown in figure 11. The image of the rod is now of the correct size and magnitude.

* See PTL Application note AN2, Calculation of Volume ratio for ECT sensors.

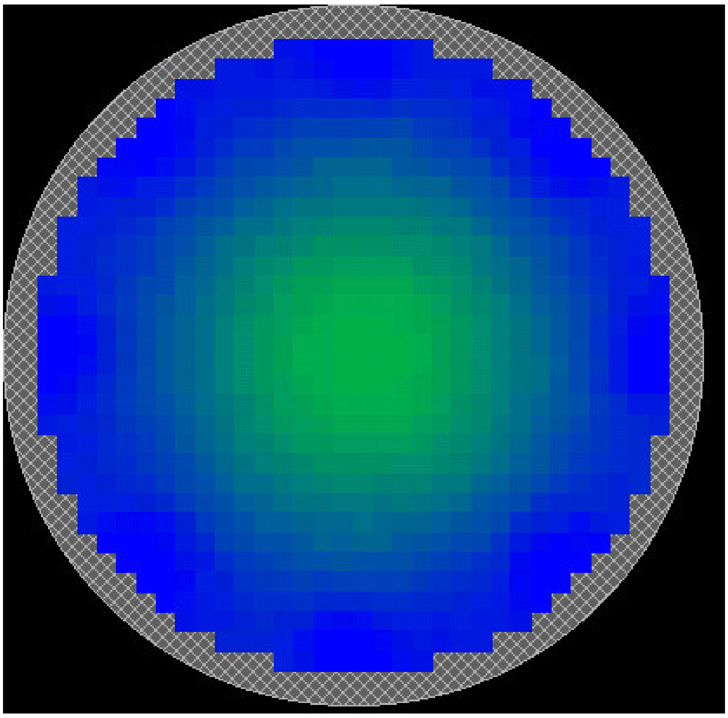

Figure 11. LBP image of circular rod

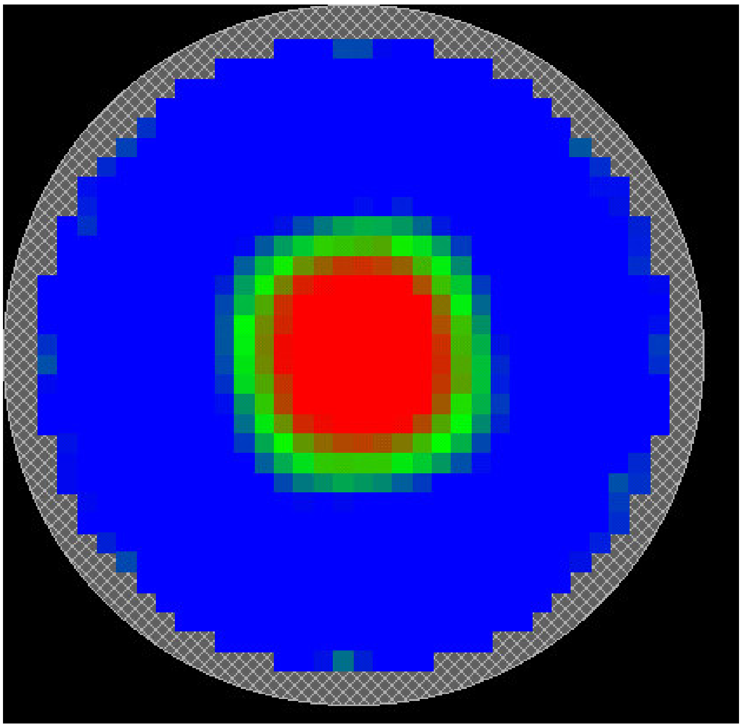

Figure 12. Same image after 50 iterations