

## **PTL APPLICATION NOTE AN2**

### **CALCULATION OF VOLUME RATIO FOR ECT SENSORS**

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#### **SUMMARY**

This note explains how the overall volume ratio (voidage), of a mixture of two dielectric materials inside an ECT sensor, and also the distribution of this voidage across the sensor, is calculated. The overall voidage can be obtained either from the measurements of the normalised capacitances between the sensor electrodes or from the permittivity distribution of the mixture, derived from these measurements. The voidage distribution is obtained from the permittivity distribution.

There are a number of possible methods which can be used to calculate the voidage and the choice of the optimum method depends on the electrical model used to describe the physical distribution of the two materials inside the sensor. For some applications, such as liquid or dense-phase mixtures, a simple parallel capacitance model can be used to obtain the voidage distribution directly from the permittivity distribution of the mixture. However, in other applications, such as fluidised beds with high levels of fluidisation, the use of a model based on capacitances in series produces better accuracy and sensitivity. A further model which combines the parallel and series models and which was developed by Maxwell in the 19th century is a useful compromise in many practical applications.

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## 1. INTRODUCTION

ECT systems can be used to obtain **images of the distribution of permittivity** inside ECT sensors for **any arbitrary mixture of different dielectric materials**.

However, an important application of ECT is viewing and measuring the spatial distribution of a **mixture of two different dielectric materials (a two-phase mixture)**. For a **two-phase mixture**, ECT can be used to measure the **spatial distribution** of the **composite permittivity** of the two materials inside the sensor. From this permittivity distribution, it is possible to obtain the distribution of the **relative concentration (volume ratio)** of the two components over the cross-section of the vessel.

A typical ECT permittivity image format uses a **square grid of 32 x 32 pixels** to display the distribution of the **normalised composite permittivity of each pixel**. For a circular sensor, 812 pixels are used to approximate the cross-section of the sensor. The **values of each pixel** represent the **normalised value of the effective permittivity of that pixel**. In the case of a **mixture of two dielectric materials**, these **permittivity values** are related to the fraction of the higher permittivity material present (the **volume ratio (or voidage)**) **at that pixel location**.

The **overall volume ratio**, which defines the **ratio of the two materials present, averaged over the volume of the sensor**, can also be obtained. The **overall volume ratio** of the materials inside the sensor at any moment in time is defined to be **the percentage of the volume of the sensor occupied by the higher permittivity material**. The volume of the sensor is the product of the cross-sectional area of the sensor and the length of the sensor measurement electrodes.

In all of the following we shall be referring to the **relative permittivity (or dielectric constant)** of materials. The **relative permittivity** of a material is its **absolute permittivity** divided by the **permittivity of free space** (or air). Hence the relative permittivity of air is 1 and typical values for other materials in solid or liquid format are polystyrene (2.5), glass (6.0) and mineral oil (2.3).

We have also used three different terms to describe the same concept, as they are all in common use. These are **volume ratio, voidage** and **concentration**, which we define to be the fraction of the higher permittivity material present in the mixture. These terms are inter-changeable in the following test.

## 2. EFFECT OF CALIBRATION PROCEDURE ON VOIDAGE CALCULATION

### 2.1 ECT SYSTEM CALIBRATION PROCEDURE

In the normal method of operation, an ECT system is calibrated by filling the sensor with the **two reference materials in turn** and by measuring the **resultant inter-electrode capacitance values** at these **two extreme values of relative permittivity**.

This situation is shown diagrammatically in figure 1, which illustrates how the measured inter-electrode capacitances change between the higher and lower values of calibration for two materials of relative permittivity  $K_L$  and  $K_H$ . For simplicity, it has been assumed that the variation is linear, although, as will be seen later, this may not always be a valid assumption.

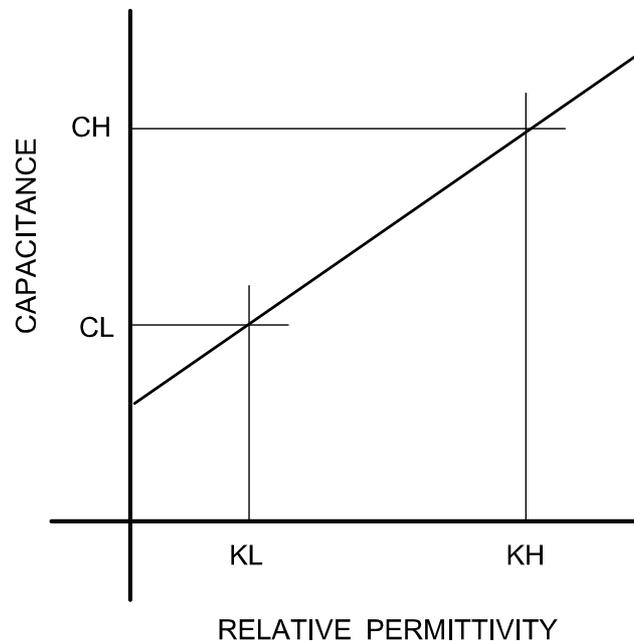


Figure 1. Relationship between the measured inter-electrode capacitances and the permittivity of the material inside the sensor.

This method of calibration defines the **two end points** of the **measurement range** for most types of ECT measurement.

## 2.2 ABSOLUTE AND RELATIVE VOIDAGES

All **voidage** values obtained from the ECT system are based on the assumption that the **voidage is 100%** when the sensor is full of the **higher permittivity material** and is **zero** when the sensor is filled with the **lower permittivity material**. Consequently, the **voidage values obtained from an ECT system are Relative Voidages**.

If the two materials used for calibration are **liquids**, then the **voidages** obtained from the ECT system will correspond nominally to the actual **absolute voidages**.

However, in many cases, one of the reference materials (the lower permittivity material) will be **air**. Air has a dielectric constant (relative permittivity) of 1, which is, by definition, the lowest possible value of dielectric constant which can exist for any real material. If the **second reference material** is in **granular or powder form**, the **upper calibration point** will be formed by a **mixture of air and the granular material**.

This will result in a **lower permittivity** for the **upper calibration point** than would be obtained by simply assuming the relative permittivity of the dielectric material in its solid form. For example for a mixture of glass beads and air, the measured permittivity of the mixture is around 3, whereas the permittivity of solid glass is approximately 6.

In this case, the **absolute voidage** for both the individual pixels and the sensor as a whole is obtained by multiplying the indicated **relative voidage** by the **actual voidage** at the upper calibration point.

For example, if the indicated **relative voidage** of a pixel is **p** and the **absolute voidage** when the sensor is full of the higher permittivity material is **f**, then **the absolute voidage of the pixel, VR**, will be given by:

$$VR = p.f \quad (2.2.1)$$

It should be noted that the permittivity or volume ratio distribution can only be obtained from the inter-electrode capacitance measurements if:

1. There are no more than 2 materials present inside the sensor.
2. The sensor has been correctly calibrated using these two materials.

### 3. NORMALISATION OF MEASUREMENT PARAMETERS

In PTL ECT systems, use is made of **normalised parameters** to represent **the inter-electrode capacitance measurements** and also the displayed values of **pixel permittivity**. The most important advantages of normalisation are:

1. It eliminates the need for the user to know the dielectric constants of the individual materials inside the sensor.
2. The widely different ranges of capacitance measurements between different combinations of electrodes are reduced to a single measurement range from 0 to 1.
3. The effect of measurement errors inside the DAM200 unit (including cross-coupling between measurement channels) is reduced.
4. ECT system calibration and operation are greatly simplified.

There are also some disadvantages to the use of normalised parameters. However, at present, the advantages of normalisation out-weigh the disadvantages and allow the ECT system to be operated in a much simpler way than would be possible if absolute values were used.

In the data capture process, the measured values of inter-electrode capacitances for the ECT sensor for each image frame and the pixel permittivity values derived from them, are normalised to lie between the values 0 and 1, where 0 corresponds to the values measured at the lower permittivity calibration point and 1 corresponds to the values measured at the upper permittivity calibration point. This is carried out using the reference data in the calibration file which is generated during the calibration process.

The following sections describe in detail how these capacitance measurements and pixel permittivities are normalised and used to calculate the volume ratio of the materials inside the ECT sensor.

### 3.1 NORMALISATION OF INTER-ELECTRODE CAPACITANCES

The inter-electrode capacitances measured at the lower calibration point ( $C_L$ ) are assigned values of 0 while the inter-electrode capacitances measured at the higher calibration point ( $C_H$ ) are assigned values of 1. This relationship is shown in graphical format below.

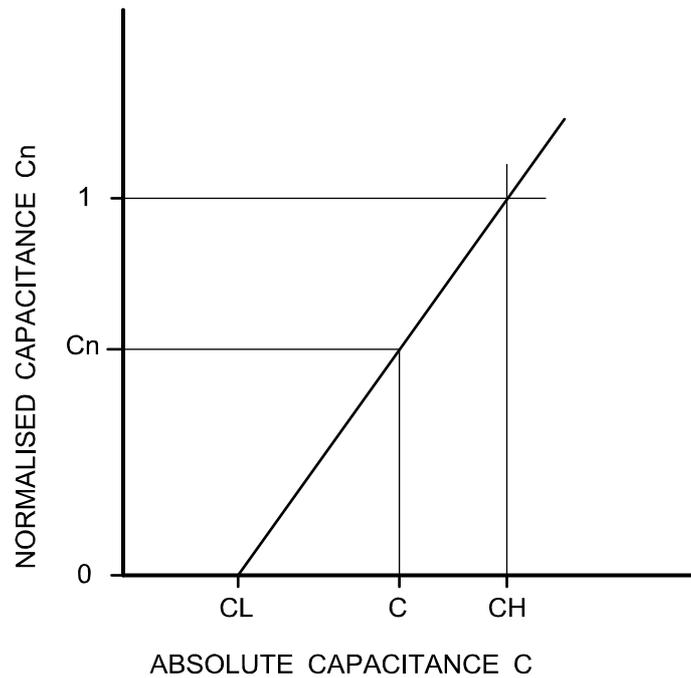


Figure 2. Normalisation of measured capacitance values

This relationship is defined by the equation:

$$C_n = (C - C_L) / (C_H - C_L) \quad (3.1.1)$$

where  $C_n$  is the set of normalised inter-electrode capacitances and  $C$  is the set of absolute capacitances measured with the sensor containing a material of arbitrary permittivity,  $C_H$  is the set of capacitances measured at the higher permittivity calibration point and  $C_L$  is the set of capacitances measured at the lower permittivity calibration point.

### 3.2 NORMALISATION OF PIXEL PERMITTIVITY VALUES

The permittivity values are normalised in a similar manner to the inter-electrode capacitances.

The permittivity values for each pixel in the ECT image for the lower permittivity calibration point ( $K_L$ ) are assigned values of 0, while the pixel permittivities in the image at the higher calibration point ( $K_H$ ) are assigned values of 1. This relationship is shown in graphical format below.

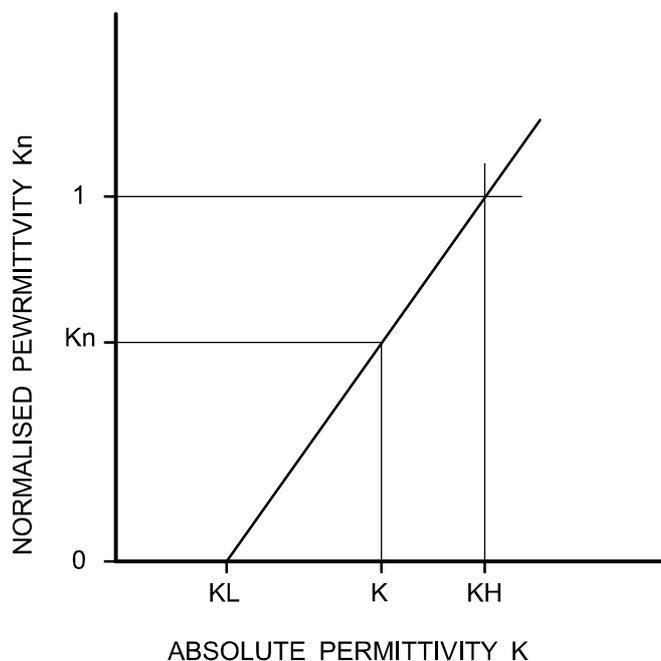


Figure 3. Normalisation of pixel permittivity values

This relationship is defined by the equation:

$$K_n = (K - K_L) / (K_H - K_L) \quad (3.2.1)$$

where  $K_n$  is the set of normalised permittivities (pixel values) when the sensor is filled with a material of permittivity  $K$ ,  $K_H$  is the effective permittivity of the material used to calibrate the sensor at the higher permittivity calibration point and  $K_L$  is the permittivity of the material used to calibrate the sensor at the lower permittivity calibration point.

### 3.3 RELATIONSHIP BETWEEN NORMALISED CAPACITANCES AND PIXEL PERMITTIVITIES

The ECT system constructs images from the normalised capacitances using Linear Back-Projection. For an ideal ECT sensor with internal electrodes and containing a dielectric material of uniform permittivity, there will be a linear relationship between the normalised inter-electrode capacitances and the resulting normalised pixel permittivity values. For example, if the sensor contains a uniform material of normalised permittivity  $K_n = P$ , the normalised inter-electrode capacitances will all have the value  $P$  and this will result in an image where each pixel also has the value  $P$ .

#### 4. CALCULATION OF RELATIVE VOIDAGE FROM ECT MEASUREMENTS

The **overall voidage** of the contents of the ECT sensor can be calculated from either the **normalised pixel values** in the reconstructed **ECT image** or from the **normalised capacitance measurements** directly. The choice is left as an option for the user to determine the method which gives results closest to the actual overall voidage. In general, the use of the pixel permittivities tends to give the most accurate results.

In the case of calculation from **image pixels**, the voidage is calculated by summing the values of the individual pixels in the ECT image for the required image frame and dividing this figure by the sum of these pixel values when the sensor is full of the higher permittivity material.

Putting this in mathematical terms,

$$VR = (1/M) \sum_{i=1}^M (P(i) / P_k) \quad (4.1)$$

where VR is the voidage, M is the total number of pixels, P(i) is the value of the ith pixel, and P<sub>k</sub> is the value of the ith pixel when the sensor is filled with the higher permittivity material (nominally 1).

In the case of calculation from the **normalised inter-electrode capacitances**, the voidage is obtained by summing all of the normalised capacitance values for one image frame and dividing these by the sum of the normalised capacitances when the sensor is filled with the higher permittivity material.

Again, putting this in mathematical terms,

$$VR = (1/N) \sum_{n=1}^N (C_n / C_k) \quad (4.2)$$

where N is the total number of electrode-pair measurements, C<sub>n</sub> are the individual electrode-pair normalised capacitances and C<sub>k</sub> are the electrode pair capacitances with the sensor full of the higher permittivity material (nominally 1).

In both of these cases, the voidage is calculated either directly or indirectly from the measurements of normalised capacitance and it is therefore necessary to consider how these capacitances or the pixel permittivity values derived from them, will be affected by the distribution of the two dielectric materials inside the ECT sensor.

This is necessary to determine whether these measurements will give an accurate assessment of the voidage. The sensor is calibrated accurately at the extreme ends of the measuring range and the voidage will be correct at these two points. However, at all points in between these two calibration points, the measured values of inter-electrode capacitances or pixel values will depend on the distribution of the dielectric materials inside the sensor.

In the next few sections we will explore how the combination of two dielectric materials determines the effective permittivity of the mixture. This is an important question as the answer will affect the accuracy of the volume ratio calculation.

## **5. EFFECTIVE PERMITTIVITY OF A MIXTURE OF TWO DIELECTRIC MATERIALS**

We are going to consider the case where we have a mixture of the two dielectric materials with which the sensor was calibrated inside the ECT sensor. Assume that these materials are uniformly mixed so that the mixture contains a fraction  $X$  of the higher permittivity material and hence  $(1 - X)$  of the lower permittivity material, ie the proportions of the two materials are constant everywhere. In this situation, we would expect the ECT image to display a uniform permittivity distribution with some constant value of effective permittivity  $K_E$ . However, we need to know how the effective permittivity  $K_E$  relates to the fraction or concentration ( $X$ ) of the higher permittivity material present in the mixture ( $X$  is also commonly referred to as the volume ratio or voidage).

The answer to this question depends on the model chosen to represent the fluid mixture. Further information about the basis of these models is given in Appendix 1.

## 5.1 THE PARALLEL CAPACITANCE MODEL

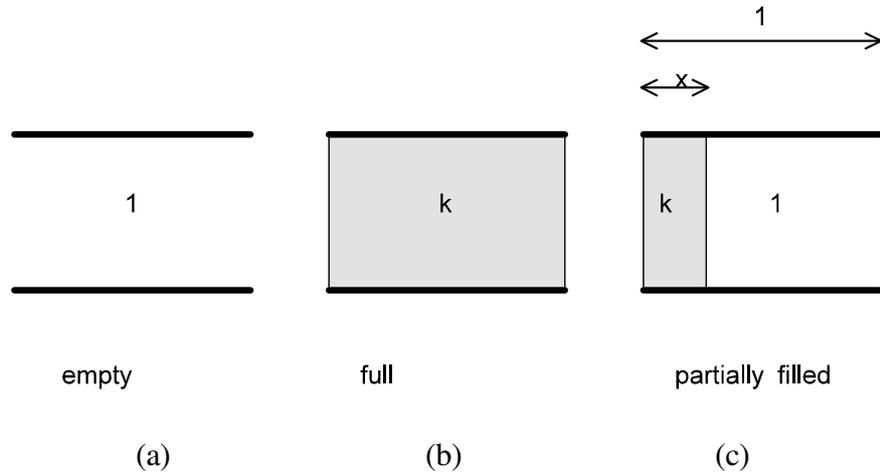


Figure 5.1 Capacitance cell containing vertically-stratified material

The simplest case assumes that the effective permittivity of the mixture can be obtained simply by summing the effects of the two components. This is known as the parallel capacitance model and corresponds to the case where there are effectively continuous bands of each dielectric material between the electrodes of the sensor as shown in figure 5.1.

In this case, the effective permittivity of the mixture  $K_E$  is given by:

$$K_E = X.K_H + (1 - X).K_L \quad (5.1.1)$$

Normalising this using equation (3.2.1), we obtain:

$$K_{EN} = (X.K_H + (1 - X).K_L - K_L) / (K_H - K_L) \quad (5.1.2)$$

giving:

$$K_{EN} = (X.K_H + K_L - X.K_L - K_L) / (K_H - K_L) \quad (5.1.3)$$

which simplifies to:

$$K_{EN} = X \quad (5.1.4)$$

So if we assume that the effective permittivities of the two materials can be found by adding their individual permittivities weighted by their relative concentrations, then the pixel values in the ECT image will correspond to the relative concentration (volume ratio)  $X$  of the higher permittivity material directly. Hence the implicit assumption made in equation 4.1, that the volume ratio distribution can be obtained directly from the permittivity distribution is correct.

The parallel capacitance model tends to be valid for densely packed materials, such as liquids, or powdered/granular materials in dense-phase processes.

More detailed information about the parallel capacitance model is given in Appendix 1.1.

## 5.2 THE SERIES CAPACITANCE MODEL

An alternative model is required for the situation where the higher permittivity material is present in dilute quantities in the mixture. This situation occurs, for example, in lean-phase fluidised beds or pneumatic conveying applications.

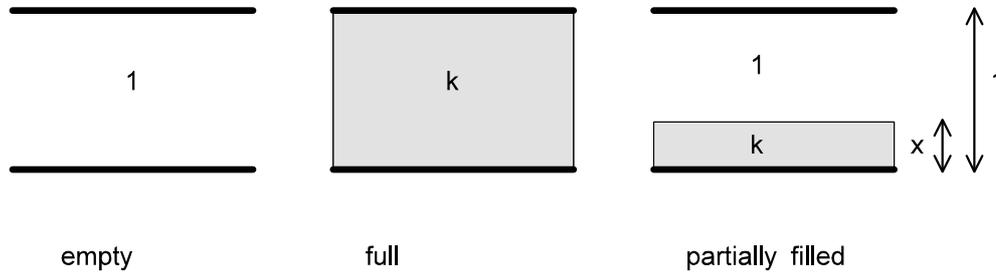


Figure 5.2. Capacitance cell containing horizontally-stratified material

The model which fits this situation is the series capacitance model, where we assume that the effective permittivity of a mixture of two materials can be found by assuming that the two materials act as two capacitors connected in series. (This model applies where neither material exists as continuous bands between the sensor electrodes). The first capacitor consists of a unit cell containing the higher permittivity material and the second capacitor consists of a similar cell containing the lower permittivity material. The cell capacitances are weighted by their permittivity values and concentrations, so that the effective permittivity of the mixture ( $K_e$ ) is obtained using the reciprocal law for two capacitors in series, ie:

$$1/C = 1/C_L + 1/C_H \quad (5.2.1)$$

These capacitances are proportional to the permittivity and inversely proportional to the concentration (see Appendix 1.2). Hence:

$$1 / K_E = (1 - X) / K_L + (X / K_H) \quad (5.2.2)$$

which simplifies to:

$$K_E = K_L \cdot K_H / (K_H + X \cdot (K_L - K_H)) \quad (5.2.3)$$

We can now calculate the normalised permittivity  $K_{EN}$  using equation 3.2.1.

$$K_{EN} = (K_L \cdot K_H / (K_H + X \cdot (K_L - K_H)) - K_L) / (K_H - K_L) \quad (5.2.4)$$

which simplifies to: 
$$K_{EN} = K_L \cdot X / ((K_H \cdot (1 - X) + K_L \cdot X)) \quad (5.2.5)$$

It is a simple task to invert this equation to obtain the volume ratio  $X$  as a function of the normalised pixel permittivity values  $K_{EN}$ :

$$X = K_{EN} \cdot K_H / [K_L + K_{EN} \cdot (K_H - K_L)] \quad (5.2.6)$$

It is clear from this equation, that volume ratio  $X$  is no longer equal to the values of normalised pixel permittivities  $K_{en}$  and equation 4.1 is no longer valid.

### 5.3 CORRECTION OF PIXEL PERMITTIVITY VALUES

Equation 5.2.6 shows that to obtain the correct values of volume ratio when applying the series capacitance model, the pixel values obtained by linear back-projection must be modified by multiplying them by a correction factor CF where:

$$CF = K_H / (K_L + K_{en}(K_H - K_L)) \quad (5.3.1)$$

This correction factor requires knowledge of the higher and lower permittivity values used for calibration, (or their ratio  $K=K_H/K_L$ ). Figure 4, which plots this correction factor for several values of  $K_H/K_L$ , shows how the indicated voidage must be modified to give the correct voidage when the series model is used.

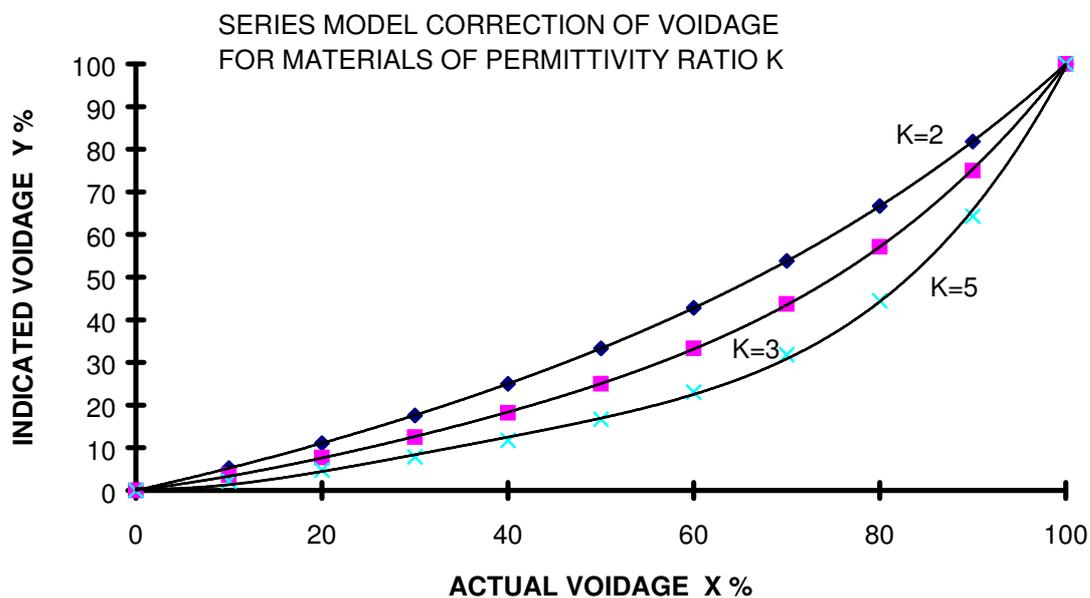


Figure 4. Correction of pixel permittivity values using the series model.

As the pixel permittivity values are obtained via a linear summation process from the normalised inter-electrode capacitance values, it is convenient, in practice, to apply this correction factor to the measured capacitances rather than to the pixel values directly. This has the added advantage that the average volume ratio of the ECT sensor contents, which can be calculated either from the pixel values or from the measured capacitances, is automatically corrected for both options.

Note that equation 5.2.6 can be re-written to give the concentration X in terms of the ratio of the high and low material permittivities (K) as follows:

$$X = K_{EN} \cdot K / [1 + K_{EN} \cdot (K - 1)] \quad (5.3.4)$$

where  $K = K_H/K_L$

## 6. OTHER PERMITTIVITY MODELS

There are a number of other models which can be used to calculate the relationship between the voidage and the effective permittivity of the material inside the sensor.

### 6.1 Maxwell Model

Details of this model are given in Appendix 2.

This yields a slightly different correction factor which, in practice is applicable to mixtures of two materials where both the parallel and series model apply in different regions of the mixture.

$$X = K_{EN} \cdot (2 + K) / [3 + K_{EN} \cdot (K - 1)] \quad (6.1.1)$$

where  $K = K_H / K_L$

### 6.2 Yang/Szuster Model

A variation on the series model, which does not rely on knowing the ratio of the permittivities of the materials used at calibration, has been developed independently by Dr W. Yang of UMIST in the UK and K. Szuster in Poland. This method effectively deduces the permittivity ratio from the calibration data.

The method is described in reference 1.

The correction factor which results from this model is given in equation 6.2.1.

$$X = K_{EN} \cdot (C_H / C_m) \quad (6.2.1)$$

where:

$C_H$  is the absolute value of the capacitance measured during calibration for the higher permittivity material.

$C_m$  is the absolute value of the measured capacitance.

## 7. REFERENCE

"An improved Normalisation Approach for Electrical Capacitance Tomography.", Yang, W.Q. and Byars, M., 1st World Congress on Industrial Process Tomography, Buxton, UK, 14-17 April 1999.

## APPENDIX 1

### A1 CAPACITANCE MODELS FOR DISTRIBUTIONS OF DIELECTRIC MATERIALS

Any mixture of two materials can be represented by sets of elemental capacitors containing one or other of the dielectrics. These elemental capacitors can be envisaged as being inter-connected either in parallel with each other (in which case the total capacitance will be the sum of the elemental capacitors) or in series with each other (in which case the reciprocal rule must be used to obtain the total capacitance). We can simplify these two situations to the case of a simple horizontal parallel plate capacitor containing two dielectric materials which may be distributed either horizontally or vertically as described in the following sections.

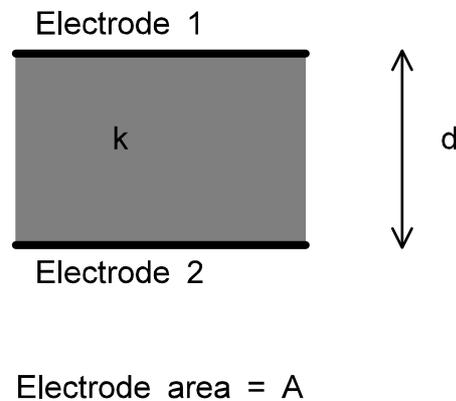


Figure A1.1. Parallel plate capacitance cell

Consider the case of the simple parallel plate capacitance cell shown in figure A1.1. The capacitance of this cell is given by the equation :

$$C = \epsilon_0 \cdot \epsilon_r \cdot A/d \quad (\text{A1.1})$$

where  $\epsilon_0$  is the permittivity of free space,  $\epsilon_r$  is the dielectric constant of the material inside the cell, A is the area of the capacitance plates and d is the spacing between the plates.

We are going to consider how the capacitance of this type of cell changes when it is filled with varying quantities of dielectric material. As the cross sectional area, the permittivity of free space and the plate spacing d will remain constant throughout, we can arbitrarily set

A.  $\epsilon_0/d = 1$  and calculate a set of relative capacitance values  $C_r$ .

In this case, equation (A1.1) simplifies to:

$$C_r = \epsilon_r \quad (\text{A1.2})$$

## A1.1 CASE 1. VERTICAL DIVISION OF MATERIALS

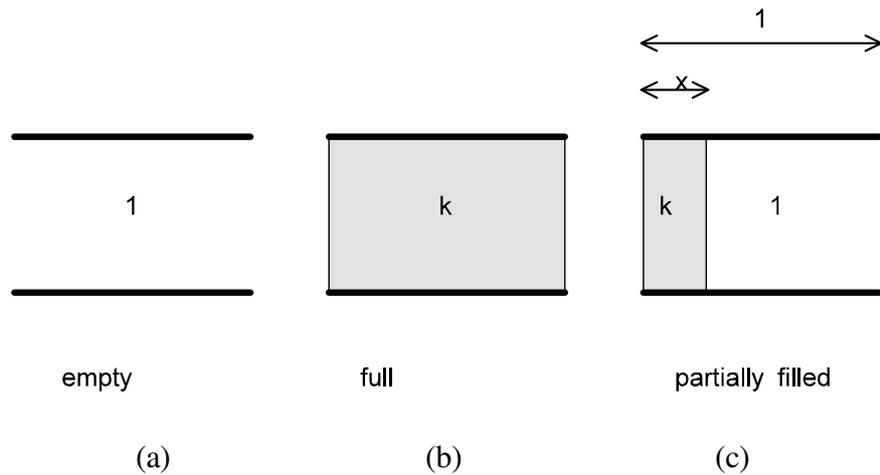


Figure A1.1.1 Capacitance cell containing vertically-stratified material

The first case to consider is that shown in the figure A1.1.1, where the cell is shown (a) empty (containing air), (b) filled with a dielectric material of relative permittivity  $k$  and (c) partially filled, with a vertical division between the dielectric material and air.

When the cell is filled with air (a),  $\epsilon_r = 1$  and equation (6.2) becomes:

$$C_{\text{air}} = 1 \quad (\text{A1.1.1})$$

When the cell is filled with dielectric material of dielectric constant  $k$  (b), the capacitance becomes:

$$C_k = k \quad (\text{A1.1.2})$$

When the cell is partially filled with dielectric material of dielectric constant  $k$  (c), the cell becomes two capacitors  $C_{\text{air}}$  and  $C_k$  in parallel, where the values of  $C_{\text{air}}$  and  $C_k$  depend on the proportion of the cell width occupied by each component.

The capacitance of parallel capacitors is simply the sum of the two individual capacitors. Hence the capacitance for the case of 2c is given by:

$$C_r = C_{\text{air}} + C_k \quad (\text{A1.1.3})$$

or 
$$C_r = 1 \cdot (1-X) + k \cdot X \quad (\text{A1.1.4})$$

giving 
$$C_r = X \cdot (k - 1) + 1 \quad (\text{A11.5})$$

In this case, the capacitance of the cell will simply increase in proportion to the width of the cell occupied by the dielectric material. Hence the **cell capacitance will increase linearly as the proportion of higher dielectric material in the cell increases** and this capacitance **will accurately represent the cell voidage**.

This model for the relationship between permittivity and capacitance is referred to as the **parallel capacitance model**.

## A1.2. CASE 2. HORIZONTAL DIVISION OF MATERIALS

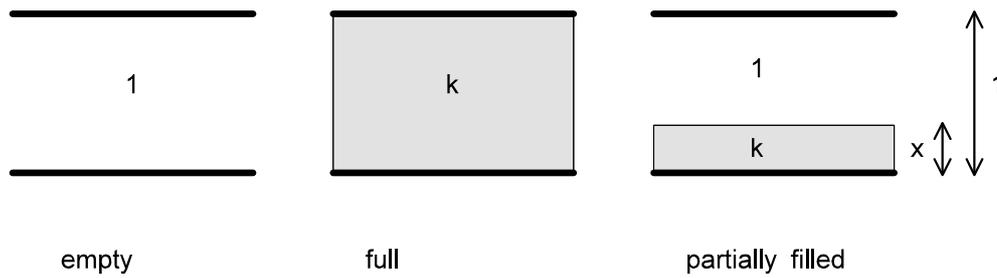


Figure A1.2.1 Capacitance cell containing horizontally-stratified material

The second case to consider is that shown in figure 5, where the cell is shown (a) empty (containing air), (b) filled with a dielectric material of relative permittivity  $k$  and (c) partially filled with a horizontal division between the dielectric material and air.

When the cell is filled with air (a),  $\epsilon_r = 1$  and equation (6.2) becomes:

$$C_{\text{air}} = 1 \quad (\text{A1.2.1})$$

When the cell is filled with dielectric material of dielectric constant  $k$  (b), the capacitance becomes:

$$C_k = k \quad (\text{A1.2.2})$$

When the cell is partially filled with dielectric material of dielectric constant  $k$  (c), the cell becomes two capacitors  $C_{\text{air}}$  and  $C_k$  in series, where the values of  $C_{\text{air}}$  and  $C_k$  depend on the proportion of the cell height occupied by each component.

Unlike the situation for capacitors connected in parallel, the capacitance of capacitors connected in series is not simply the sum of the two individual capacitors but is obtained by adding up their reciprocals. Hence the capacitance for the case of 2c is given by:

$$1/C_r = 1/C_{\text{air}} + 1/C_k \quad (\text{A1.2.3})$$

Now  $C_{\text{air}} = 1/(1 - X) \quad (\text{A1.2.4})$

and  $C_k = k/X \quad (\text{A1.2.5})$

Hence  $1/C_r = (1 - X) + X/k \quad (\text{A1.2.6})$

Giving  $C_r = k/(k(1 - X) + X) \quad (\text{A1.2.7})$

In this case, **the capacitance of the cell increases in a non-linear manner as the proportion of the cell occupied by the dielectric material increases. Moreover, the rate of increase depends on the dielectric constant of the material.**

This model for the relationship between permittivity and capacitance is referred to as the **series capacitance model**. In many cases (such as fluidised beds), the series capacitance model is a more realistic accurate representation of the physical model than the parallel capacitance model.

## APPENDIX 2

### MAXWELL PERMITTIVITY MODEL

This model is based on the work of Maxwell in the 19th Century. Maxwell showed that, for a homogeneous mixture containing small spheres of equal diameter of a higher permittivity material  $\epsilon_2$  distributed within a lower permittivity material  $\epsilon_1$ , the effective permittivity of the material is given by:

$$\epsilon_m = \epsilon_1 \cdot \frac{2 \cdot \epsilon_1 + \epsilon_2 - 2 \cdot X \cdot (\epsilon_1 - \epsilon_2)}{2 \cdot \epsilon_1 + \epsilon_2 + X \cdot (\epsilon_1 - \epsilon_2)} \quad (\text{A2.1})$$

As the measured capacitances will be proportional to the effective permittivity  $\epsilon_m$ , we can re-write equation A2.1 as follows:

$$C = \frac{2 + K - 2 \cdot X \cdot (1 - K)}{2 + K + X \cdot (1 - K)} \quad (\text{A2.2})$$

where  $K = \epsilon_2 / \epsilon_1$

We can now repeat the process outlined in Appendix 1 and section 5 to calculate the effective normalised permittivity  $K_{EN}$  (which is equal to the normalised capacitance  $C_n$ ), in terms of the voidage  $X$ , which yields:

$$K_{EN} = 3 \cdot X / [2 + k + X \cdot (1 - K)] \quad (\text{A2.3})$$

As before, we can invert this equation to obtain the actual voidage  $x$  in terms of the pixel permittivity  $K_{en}$  as follows:

$$X = K_{EN} \cdot (2 + K) / [3 + K_{EN} \cdot (K - 1)] \quad (\text{A2.4})$$

This yields an alternative correction factor for the measured inter-electrode capacitances.